1 Komplexe Zahlen:

komplexe Einheit: $i^2 = -1$, also $i^{2m} = (-1)^m$ und $i^{2m+1} = (-1)^m \cdot i$

komplexe Zahl: $z = a + ib$; mit $a = Re(z)$ und $b = Im(z)$

konjugierte komplexe Zahl: $z^* = \overline{z} = a - ib$ mit $z \cdot z^* = |z|^2$

Polarform: $z = a + ib = |z| \cdot (\cos\varphi + i \sin\varphi) = |z| \cdot E(\varphi)$

mit $\varphi = \arg(z)$ und $\tan\varphi = \frac{b}{a}$

$\Rightarrow a = |z| \cos\varphi$ und $b = |z| \sin\varphi$

Satz von MOIVRE: $[E(\varphi)]^n = E(n\varphi)$ für alle $\varphi \in \mathbb{R}, n \in \mathbb{N}$

EULER**'sche Formel:** $E(\varphi) = e^{i\varphi}$ also: $z = a + ib = |z| \cdot e^{i\varphi}$

2 Logarithmen

$\log_b(u \cdot v) = \log_b u + \log_b v$

$\log_b\left(\frac{u}{v}\right) = \log_b u - \log_b v$

$\log_b(u^z) = z \cdot \log_b u$

$\log_b(\sqrt[z]{u}) = \frac{1}{z} \cdot \log_b u$

$\log_c a = \frac{\log_b a}{\log_b c}$

3 Exponentialfunktion

$\exp(x) = e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ mit $e = 2,718281828...$ und $x \in \mathbb{R}$

$e^{i \cdot \varphi} = \cos(\varphi) + i \cdot \sin(\varphi)$

$e^{-i \cdot \varphi} = \cos(\varphi) - i \cdot \sin(\varphi)$

$e^z = e^a(\cos b + i \sin b)$

$z = (a + ib) \in \mathbb{C}$

4 Trigonometrische Funktionen

4.1 Definition

$x \in \mathbb{R}$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$$

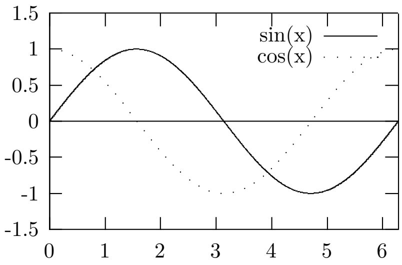
$$\cosh(x) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1} = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

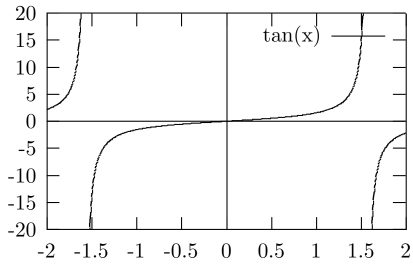
$$\sinh(x) = \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n} = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

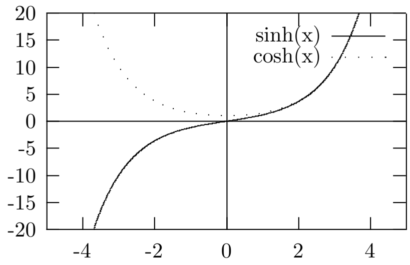
$z = (a + ib) \in \mathbb{C}$

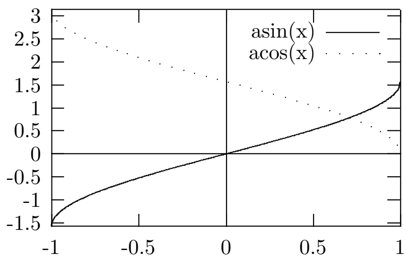
$$\cos(z) = \frac{\left(e^b + e^{-b}\right) cos(a) + i\left(e^{-b} - e^b\right)\sin(a)}{2}$$

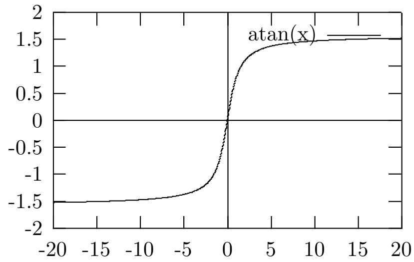
$$\sin(z) = \frac{\left(e^{-b} - e^b\right) cos(a) + i\left(e^{-b} + e^b\right)\sin(a)}{2i}$$











φ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\varphi$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1	0	-1
$\cos\varphi$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	-1	0
$\tan\varphi$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	nicht def.	0	nicht def.

4.2 (Additions-)Theoreme

$$\sin^2\varphi + \cos^2\varphi = 1$$

$$\tan\varphi\cot\varphi = 1 \quad (\varphi \neq k \cdot 90^\circ)$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\cos\alpha\sin\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\sin^2\alpha = \frac{1}{2}(1-\cos 2\alpha)$$

$$\cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha)$$

$$\sin^3\alpha = \frac{1}{4}(3\sin\alpha - \sin 3\alpha)$$

$$\cos^3\alpha = \frac{1}{4}(\cos 3\alpha - 3\cos\alpha)$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$$

$$2\sin\alpha\cos\beta = \sin(\alpha-\beta) + \sin(\alpha+\beta)$$

$$\cosh^2\varphi - \sinh^2\varphi = 1$$

$$\sinh(\alpha\pm\beta) = \sinh\alpha\cosh\beta \pm \cosh\alpha\sinh\beta$$

$$\cosh(\alpha\pm\beta) = \cosh\alpha\cosh\beta \pm \sinh\alpha\sinh\beta$$

4.3 Zusammenhang zwischen hyperb. und trigon. Funktionen

$z \in \mathbb{C}$

$\cos z = -i \cosh iz$

$\sin z = -i \sinh iz$

$\tan z = -i \tanh iz$

$\cosh z = -i \cos iz$

$\sinh z = -i \sin iz$

$\tanh z = -i \tan iz$

5 Umformunegn in $\mathbb{R}$

5.1 Binome, Trinome

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

$a^2 + b^2$ in $\mathbb{R}$ nicht zerlegbar

$a^2 - b^2 = (a+b)(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1}$

5.2 Fakultäten, Binomialkoeffizient ...

Fakultät: $n! = 1 \cdot 2 \cdot 3 \cdot ... \cdot n$ mit $n \in \mathbb{N}$; $0! = 1$; $1! = 1$

Binomialkoeffizient: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ für $k \leq n$

$\binom{n}{k} = 0$ für $k > n$

Binomischer Satz: $(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + ... + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$

5.3 wichtige Ungleichungen ...

Dreiecksungleichung: $|a+b| \leq |a| + |b|$

$|a-b| \geq |a| - |b|$

Ungleichung von Bernoulli: $(1+x)^n \geq 1 + nx$ für $-1 < x$; $n \in \mathbb{N}$

Young**'sche Ungleichung:** $2\,|ab| \leq \epsilon a^2 + \epsilon^{-1} b^2$ für $\epsilon \in \mathbb{R}_+$

6 Entwicklungen ...

TAYLOR-**Entwicklung um** x_0**:** $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$

TAYLOR-**Entwicklung um** $x_0 = 0$**:** $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}(x)^k$

7 Differentiation

Produktregel: $f(x) = u(x) \cdot v(x) \quad \Rightarrow \quad f'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$

Quotientenregel: $f(x) = \frac{u(x)}{v(x)} \quad \Rightarrow \quad f'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{[v(x)]^2}$

Kettenregel: $h'(x) = (g \circ f)'(x) = g'(f(x)) \cdot f'(x)$

Ableitungen der Grundfunktionen:

$f(x) = x^n,\ (n \in \mathbb{R})$	$f'(x) = n \cdot x^{n-1}$		
$f(x) = \sin x$	$f'(x) = \cos x$	$f(x) = \cos x$	$f'(x) = -\sin x$
$f(x) = \tan x$	$f'(x) = \frac{1}{\cos^2 x}$	$f(x) = \cot x$	$f'(x) = -\frac{1}{\sin^2 x}$
$f(x) = \arcsin x$	$f'(x) = \frac{1}{\sqrt{1-x^2}}$	$f(x) = \arccos x$	$f'(x) = -\frac{1}{\sqrt{1-x^2}}$
$f(x) = \arctan x$	$f'(x) = \frac{1}{1+x^2}$	$f(x) = arccotx$	$f'(x) = -\frac{1}{1+x^2}$
$f(x) = a^x,\ (a > 0)$	$f'(x) = a^x \cdot \ln a$	$f(x) = e^x$	$f'(x) = e^x$
$f(x) = \ln_b x, \begin{cases} b > 0 \\ b \neq 1 \end{cases}$	$f'(x) = \frac{1}{x \cdot \ln b}$	$f(x) = \ln x$	$f'(x) = \frac{1}{x}$

8 Integration

9 Stochastik, Statistik und Fehlerrechnung

Mittelwert: $\overline{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$

Standardabweichung (mittlerer Fehler): $\sigma_E = S_E(X) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}$

mittlerer Fehler des Mittelwertes: $\sigma_M = S_M(X) = \frac{S_E(X)}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n(n-1)}}$

Fehlerfortpflanzungsgesetz: $\Delta z = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2 + \ldots} \quad \text{mit } z = f(x, y, \ldots)$

Funktion	**absoluter Fehler**	**relativer Fehler**
$z = a \cdot x; \quad a = const$	$\Delta z = a \cdot \Delta x$	$\frac{\Delta z}{z} = \frac{\Delta x}{x}$
$z = x \pm y$	$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$	
$z = x \cdot y$	$\Delta z = \sqrt{(y \cdot \Delta x)^2 + (x \cdot \Delta y)^2}$	$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
$z = \frac{x}{y}$	$\Delta z = \sqrt{\left(\frac{\Delta x}{y}\right)^2 + \left(\frac{x}{y^2}\Delta y\right)}$	$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$

10 schönste Formel der Mathematik

$$e^{i\pi} + 1 = 0$$

Diese Formel enthält alle wichtigen Zahlen der Mathematik ... und nur diese!