$\omega = \frac{2\pi}{T} = 2\pi\nu$, $k = \frac{2\pi}{\lambda}$, $c = \frac{\omega}{k}$, $\lambda = \frac{c}{\nu}$, $\lambda_{DB} = \frac{h}{mv} = \frac{h}{p}$, $v_{gr} = \frac{d\omega}{dk} = \frac{\hbar k}{m}$

Photon: $\vec{p} = \hbar \vec{k}$, $E = \hbar\omega = \frac{hc}{\lambda}$, $m = \frac{h\nu}{c^2}$

$$\boxed{e^{\hat{A}} = \sum_n \frac{1}{n!} A^n}$$

Schrödinger-Gl.: $\hat{H}|\psi\rangle = E \cdot |\psi\rangle$; $i\hbar \frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$

Zeitentwicklung: $|\psi(t)\rangle = e^{-i\frac{\hat{H}}{\hbar}(t-t_0)}|\psi(t=0)\rangle = \underbrace{U(t,t_0)}_{\text{unitärer Operator}}|\psi(t=0)\rangle$

$\{\varphi_n\}$-Darstellung: $U(t,t_0) = \sum_n e^{-iE_n(t-t_0)/\hbar}|\varphi_n\rangle\langle\varphi_n|$

Erhaltungsgrößen $\frac{d}{dt}\langle\hat{A}\rangle = \frac{1}{i\hbar}\langle[\hat{A},\hat{H}]\rangle + \left\langle\frac{\partial\hat{A}}{\partial t}\right\rangle$;

$$\boxed{L = X \times P = \begin{pmatrix} YP_z - ZP_y \\ ZP_x - X P_z \\ XP_y - YP_x \end{pmatrix}}$$

Unschärferelation: $\Delta\hat{A} \cdot \Delta\hat{B} \geq \frac{1}{2}|\langle[\hat{A},\hat{B}]\rangle|$

Heisenberg-Gleichung: $\frac{d}{dt}\hat{A}_H = \frac{i}{\hbar}[\hat{H},\hat{A}_H] + \frac{\partial}{\partial t}\hat{A}_H$, $|\psi\rangle_H = e^{i\hat{H}t/\hbar}|\psi(t)\rangle_S = |\psi_0\rangle_S$

Störungsrechnung:

$$E_n = E_n^0 + \langle\psi_n|\hat{W}|\psi_n\rangle + \sum_{p\neq n}\sum_i \frac{|\langle\psi_p^i|\hat{W}|\psi_n\rangle|^2}{E_n^0 - E_p^0}$$

$$|\psi_n^{(korr)}\rangle = |\psi_n\rangle + \sum_{p\neq n}\sum_i \frac{\langle\psi_p^i|\hat{W}|\psi_n\rangle}{E_n^0 - E_p^0}$$

Ritz'sches Verfahren: $\langle\hat{H}\rangle(\alpha) = \frac{\langle\psi(\alpha)|\hat{H}|\psi(\alpha)\rangle}{\langle\psi(\alpha)|\psi(\alpha)\rangle} \geq E_0$ ~~um~~ bzgl. α minimieren

Ort-/Impuls: $\psi(x) = \langle x|\psi\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int e^{ipx/\hbar}\hat{\psi}(p)\,dp$; $\hat{\psi}(p) = \langle p|\psi\rangle = \int\frac{dx}{\sqrt{2\pi\hbar}}e^{-ipx/\hbar}\psi(x)$

Wahrsch.-Strom: $\rho(\vec{x},t) = |\psi(x,t)|^2$ $\frac{\partial}{\partial t}\rho + \vec{\nabla}\vec{j} = 0$ $\vec{j} = \frac{i\hbar}{2m}[\psi\cdot\vec{\nabla}\psi^* - \psi^*\cdot\vec{\nabla}\psi]$

δ-Pot.: $\lim_{x>x_0}\psi' - \lim_{x<x_0}\psi' = -2D\psi(x_0)$, $V(x) = V_1(x) - \frac{\hbar^2}{m} D\cdot\delta(x-x_0)$, $D\in\mathbb{R}$, V_1 stetig

Pauli-M.: $\tau_1 = \begin{pmatrix}0&1\\1&0\end{pmatrix}$, $\tau_2 = \begin{pmatrix}0&-i\\i&0\end{pmatrix}$, $\tau_3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $\hat{\vec{S}} = \frac{\hbar}{2}\vec{\tau}$, $\tau_i^2 = \mathbb{1}$, $[\hat{S}_i,\hat{S}_j] = i\hbar\varepsilon_{ijk}\hat{S}_k$

Operatoren: $\hat{X} = x$, $\hat{P}_x = -i\hbar\frac{\partial}{\partial x}$, $\hat{\vec{P}} = -i\hbar\vec{\nabla}$, $\hat{\vec{L}} = \vec{Q}\times\vec{P}$, $\hat{n} = a^+a$

$[P_i,Q_j] = -i\hbar\delta_{ij}$, $[Q_i,Q_j] = 0$, $[P_i,P_j] = 0$, $[L_i,L_j] = i\hbar\varepsilon_{ijk}L_k$, $[L_i,\vec{L}^2] = 0$

$[a_i,a_j^+] = \delta_{ij}$; $[a,a^+] = aa^+ - a^+a = aa^+ - \hat{n} = 1$

Oszi: $\hat{H} = \frac{P^2}{2m} + \frac{m\omega^2}{2}\hat{Q}^2$, $\hat{a} = \frac{1}{\sqrt{2\hbar}}\left(\sqrt{m\omega}\,\hat{Q} + \frac{i}{\sqrt{m\omega}}\hat{P}\right)$, $\hat{a}^+ = \frac{1}{\sqrt{2\hbar}}\left(\sqrt{m\omega}\,\hat{Q} - \frac{i}{\sqrt{m\omega}}\hat{P}\right)$

$\hat{a}^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$ $\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$ $E_n = (n+\frac{1}{2})\hbar\omega$ $\psi_0 = \langle x|\psi_0\rangle = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}e^{-\frac{1}{2}\frac{m\omega^2}{\hbar}x^2}$

$$H_n = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}\cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)e^{-\frac{1}{2}\frac{m\omega}{\hbar}x^2}$$

Drehimpuls: $\vec{L}^2|\psi\rangle = l(l+1)\hbar^2|\psi\rangle$ $L_z|\psi\rangle = m\hbar|\psi\rangle$, $-l \leq m \leq l$

$L_+ = L_x + iL_y$, $L_- = L_x - iL_y$, $[L_z,L_+] = \hbar L_+$, $[L_z,L_-] = -\hbar L_-$, $[L_+,L_-] = 2\hbar L_z$

$[L^2,L_+] = [L^2,L_-] = [L^2,L_z] = 0$

1s-Niveau: $|1,0,0\rangle = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$

2s-Niveau: $|2,0,0\rangle = \frac{1}{\sqrt{8\pi a_0^3}}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}$

2p-Niveau: $|2,1,1\rangle = -\frac{1}{8\sqrt{\pi a_0^3}}\frac{r}{a_0}\cdot e^{-r/2a_0}\cdot\sin(\theta)\cdot e^{i\varphi}$

$|2,1,0\rangle = -\frac{1}{4\sqrt{2\pi a_0^3}}\frac{r}{a_0}\cdot e^{-r/2a_0}\cdot\cos(\theta)$

$|2,1,-1\rangle = -\frac{1}{8\sqrt{\pi a_0^3}}\frac{r}{a_0}\cdot e^{-r/2a_0}\cdot\sin(\theta)\cdot e^{-i\varphi}$

$$L_-|n,l,m>-l\rangle = \hbar\sqrt{l(l+1)-m(m-1)}\,|n,l,m-1\rangle$$
$$L_+|n,l,m<l\rangle = \hbar\sqrt{l(l+1)-m(m+1)}\,|n,l,m+1\rangle$$

Wasserstoff: $E_n = \frac{1}{n^2}\frac{\mu e^4}{2\hbar^2}$, $r_n = n^2 a_0$, $a_0 = \frac{\hbar^2}{\mu e^2}$, $\mu = \frac{m_1 m_2}{m_1+m_2}$, $l < n$

$$\psi_{nlm}(r,\theta,\phi) = \psi_{nl}(r)Y_{lm}(\theta,\phi)$$

kennengelernt. Dabei ist

$$\psi_{nl}(r) = \frac{2}{n^2}\sqrt{\frac{(n-l-1)!}{((n+l)!)^3}}a^{-3/2}\left(\frac{2r}{na}\right)^l e^{-\frac{r}{na}}L_{n-l-1}^{2l+1}\left(\frac{2r}{na}\right)$$

mit $a := \frac{\hbar}{mZe^2}$. Dabei sind $L_p^k(\rho)$ die Laguerrepolynome, die gegeben sind durch

$$L_p^k(\rho) = \frac{\Gamma(p+k+1)}{p!}\frac{e^\rho}{\rho^k}\frac{d^p}{d\rho^p}\left(\rho^{p+k}e^{-\rho}\right)$$

Skalarprodukt $\langle\psi|\varphi\rangle = \int \psi^* \varphi \, d^3x$, $\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$

$\langle\psi|\lambda\phi\rangle = \lambda\langle\psi|\phi\rangle$, $\langle\lambda\psi|\phi\rangle = \lambda^*\langle\psi|\phi\rangle$

$|\langle\psi|\phi\rangle| \leq \sqrt{\langle\psi|\psi\rangle} \cdot \sqrt{\langle\phi|\phi\rangle}$ (Schwarz'sche Ugl.)

$|\psi\rangle = \sum c_i |i\rangle$, $|\phi\rangle = \sum b_i |i\rangle \Rightarrow \langle\phi|\psi\rangle = \sum b_i^* c_i$

Vollständigkeitsrelation $\sum_i u_i(\vec{x})\, u_i^*(\vec{x}') = \delta(\vec{x} - \vec{x}')$; $\sum_i |u_i\rangle\langle u_i| = \mathbb{1}$

Projektor: $P_\psi = |\psi\rangle\langle\psi|$, $\boxed{P_\psi^2 = P_\psi}$

Hermite'sche Operatoren: $(\hat{A}\,\hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger$

$\hat{A}^\dagger = \hat{A}$, EW reell , orthogonale EV , $\langle\hat{A}\phi|\psi\rangle = \langle\phi|\hat{A}\psi\rangle$, $\langle\psi|\hat{A}|\phi\rangle = \langle\phi|\hat{A}|\psi\rangle^*$

Unitäre Operatoren: $\hat{A}^\dagger = \hat{A}^{-1}$; $\hat{A}^\dagger\hat{A} = \mathbb{1}$

Darstellungen: $|\psi\rangle \hat{=} \begin{pmatrix} \langle u_1|\psi\rangle \\ \vdots \end{pmatrix}$ $\hat{A} \hat{=} \begin{pmatrix} A_{11} & A_{12} & \cdots \\ A_{21} & \ddots & \\ \vdots & & \end{pmatrix}$ mit $A_{ij} = \langle u_i|\hat{A}|u_j\rangle$

Basiswechsel: $\{u_i\} \to \{t_i\}$; $S_{ik} = \langle u_i|t_k\rangle$ $(S^\dagger)_{ki} = \langle t_k|u_i\rangle$, $\hat{S}$ unitär $(\hat{S}^\dagger = \hat{S}^{-1})$

$\psi_t = S^\dagger \psi_u$ $\langle t_k|\psi\rangle = \sum_i \langle t_k|u_i\rangle\langle u_i|\psi\rangle = \sum S_{ki}^\dagger \langle u_i|\psi\rangle$

$\psi_u = S\,\psi_t$ $\langle u_i|\psi\rangle = \sum S_{ik}\langle t_k|\psi\rangle$ $A_t = S^\dagger A S$ $\langle t_k|A|t_l\rangle = \sum_{ij} \langle t_k|u_i\rangle\langle u_i|A|u_j\rangle\langle u_j|t_l\rangle$

Erwartungswert: $\langle\hat{A}\rangle_\psi = \dfrac{\langle\psi|\hat{A}|\psi\rangle}{\langle\psi|\psi\rangle}$ **Standardabw.** $\Delta\hat{A} = \sqrt{\langle(\hat{A} - \langle\hat{A}\rangle)^2\rangle} = \sqrt{\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2}$

Integrale: $\int_{-\infty}^{\infty} e^{-c(x-x_0)^2} dx = \sqrt{\frac{\pi}{c}}$, $\frac{2}{\sqrt{\pi}}\int_0^1 e^{-x^2} dx = \mathrm{erf}(1) = 0{,}8427$, $\int \frac{1}{x} dx = \ln|x|$, $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$

$\int f \cdot g' = fg - \int f'g$ $\int_0^\infty x^n e^{-ax^2} dx = \frac{k!}{2a^{k+1}}$ (n ungerade), $\int_0^\infty x^2 e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{2a}$ $(a > 0)$

ε-Tensor $\varepsilon_{123} = \varepsilon_{312} = \varepsilon_{231} = 1$; $\varepsilon_{321} = \varepsilon_{213} = \varepsilon_{132} = 1$, sonst 0 $\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

Fourier: $\hat{\phi}(k) = \int_{-\infty}^{\infty} dx\, e^{-ikx} \phi(x)$, $\phi(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikx} \hat{\phi}(k)$, $\mathcal{F}[\phi(x-a); k] = e^{-ika}\mathcal{F}[\phi(x), k]$

δ-Fkt.: $\delta(x-a) = \frac{1}{2\pi}\int e^{ik(x-a)} dk$, $\delta(-x) = \delta(x)$, $x \cdot \delta(x) = 0$, $x \cdot \delta'(x) = -\delta(x)$,

$\delta(kx) = \frac{1}{b}\delta(x)$ $(b > 0)$, $\delta(t) = \frac{d}{dt}\Theta(t)$ $\mathcal{F}[\delta(x); k] = 1/2\pi$

Kugelflächen:

$P_l^{-m} = (-1)^m \frac{(l-m)!}{(l+m)!} P_l^m(x)$

$$P_n(x) = \frac{1}{2^n \cdot n!} \cdot \frac{d^n (x^2-1)^n}{dx^n} \qquad P_l^m(x) = (-1)^m (1-x^2)^{\frac{m}{2}} \cdot \frac{d^m P_l(x)}{dx^m}$$

$$\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} Y_{l'}^{m'*}(\theta,\phi) \cdot Y_l^m(\theta\phi)\, \sin\theta \, d\theta d\phi = \delta_{l'l}\delta_{m'm}$$

Kugelkoordinaten folgende Gestalt: $(r, \theta, \phi) \to (-r, \pi - \theta, \pi + \phi)$ verhalten sich die Kugelflächenfunktionen wie folgt:

$$Y_l^m(\pi - \theta, \pi + \phi) = (-1)^l \cdot Y_l^m(\theta, \phi)$$

Die Kugelflächenfunktionen $Y_l^m(\theta, \phi)$ sind definiert als:

$$Y_l^m(\theta, \phi) = \frac{1}{2\pi}\sqrt{\frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}} \cdot P_l^m(\cos\theta) e^{im\phi}.$$

Dabei sind P_l^m die zugeordneten Legendre-Polynome. Die ersten Kugelflächenfunktionen

$Y_0^0(\theta,\phi) = \frac{1}{\sqrt{4\pi}}$,

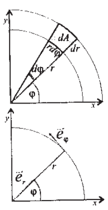
$Y_1^0(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta$, $Y_1^1(\theta,\phi) = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$

$Y_2^0(\theta,\phi) = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$,

$Y_2^1(\theta,\phi) = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$, $Y_2^2(\theta,\phi) = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{2i\phi}$

$[Y_l^m(\theta,\phi)]^* = (-1)^m \cdot Y_l^{-m}(\theta,\phi)$

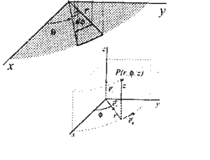
Hermite: $H_n = (-1)^n e^{x^2/2} \frac{d^n}{dx^n}\left(e^{-x^2/2}\right)$, $H_0 = 1$, $H_1 = 2x$, $H_2 = 4x^2 - 2$, $H_3 = 8x^3 - 12x$



$x = r \cdot \cos\varphi$
$y = r \cdot \sin\varphi$

$\vec{e}_r = (\cos\phi, \sin\phi)$
$\vec{e}_\phi = (-\sin\phi, \cos\phi)$

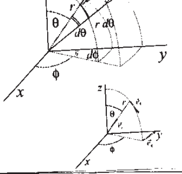
$dA = r \cdot dr\, d\varphi$
$d\vec{s} = (dr, r\, d\varphi)$



$x = r \cdot \cos\phi$
$y = r \cdot \sin\phi$
$z = z$

$\vec{e}_r = (\cos\phi, \sin\phi, 0)$
$\vec{e}_\phi = (-\sin\phi, \cos\phi, 0)$
$\vec{e}_z = (0, 0, 1)$

$dV = r \cdot dr\, d\phi\, dz$
$d\vec{A} = \vec{e}_r \cdot (r \cdot d\phi\, dz)$
$d\vec{s} = (dr, r\, d\phi, dz)$



$x = r \cdot \sin\theta \cdot \cos\phi$
$y = r \cdot \sin\theta \cdot \sin\phi$
$z = r \cdot \cos\theta$

$\vec{e}_r = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$
$\vec{e}_\theta = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)$
$\vec{e}_\phi = (-\sin\phi, \cos\phi, 0)$

$d\Omega = \sin\theta\, d\theta\, d\phi$
$dV = r^2 \sin\theta \cdot dr\, d\theta\, d\phi$
$d\vec{A} = \vec{e}_r \cdot (r^2 \sin\theta \cdot d\theta\, d\phi)$
$d\vec{s} = (dr, r \cdot d\theta, r \cdot \sin\theta \cdot d\phi)$